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COINTEGRATION TESTS BASED ON RECORD COUNTING STATISTICS

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Keywords and Phrases: Cointegrating relationships, records, counting process, co-records, ranges, monotonic nonlinearities, structural breaks, additive outliers, robustness, invariance.

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Cointegration Tests Based on Record Counting Statistics[¤]

by

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Abstract

This paper presents of number of cointegration tests that exploit the statistical properties of the records from the original time series variables. We prove their consistency and obtain their asymptotic null distributions. Among the advantages of this novel methodology, the new tests are invariant with respect to the individual series' variances and also with respect to monotonic transformations applied to these series. In addition, these tests are robust against the presence of level breaks as long as the number of these breaks increases slowly enough with the sample size. Finally, an alternative scheme is proposed to deal with additive outliers, which prevent them from causing size distortions.

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1 INTRODUCTION

1.1 Generalities

Many real-world time series are not stationary in their levels and exhibit some type of stochastic trend. Such time series are often called integrated as they require successive differencing to yield a stationary and invertible AutoRegressive Moving-Average (ARMA) representation. Formally, a time series x_t is said to be integrated of order d, or briefly, $x_t \sim I(d)$ if d is the smallest number of times [2] that x_t has to be differenced so as to have an ARMA or $I(0)$ representation. That is, if we define the first difference operator Δ as $\Delta x_t = x_t - x_{t-1}$ and by recursion the i-th difference operator as $\Delta^i x_t = \Delta^{i-1} x_t - \Delta^{i-1} x_{t-1}$; then

$$x_t \sim I(d) \ , \ d = \min \left\{ i \in Z^+ ; \ \Delta^i x_t \sim I(0) \right\} :$$

The definition of an $I(0)$ process can also be extended to include a much wider class of time series models by just requiring the time series to satisfy a functional central limit theory, as in Davidson (1998). The presence of stochastic trends prompts some technical problems for the analyst, one of which occurs when testing a theory establishing that a given variable is formally linked to another in the long run. The problem stems from the fact that unrelated series of this type have nonsense spurious regressions (Granger and Newbold, 1974; Phillips, 1986; and Phillips and Durlauf, 1986). This means that an empirical relationship is found more often than it should; a problem which does not dissappear with increasing

[2] The case of fractional differencing is out of the scope of this paper.

sample size. Therefore such theories cannot be tested empirically using the standard regression procedures based on the examination of the determination or R^2 coefficient. When a couple of univariate time series, x_t and y_t; have a long-run relationship they cannot wander far from each other and therefore deviations from this long run path must be stationary. The concept of cointegration was coined by Granger (1981) to describe this property. Two time series are said to be (linearly) cointegrated if their long-run relationship is significant and linear, or in other words, if there is a nonzero real number α so that $\varepsilon_t = y_t - \alpha x_t$ is an $I(0)$ process. For this to be possible both time series must be integrated of the same order d. A most interesting case arises when $d = 1$, since many economic time series seem to have a unit root. Rather than inspecting the quality of the adjustment in the regression between two $I(1)$ processes, a test of unit roots such as the standard Dickey-Fuller (DF) and the Augmented Dickey-Fuller (ADF) tests (Dickey and Fuller, 1979), or the related Phillips-Perron (PP) test (Phillips and Perron, 1988), is often applied to the OLS residuals from that static regression of the variables. Such devices are known as residual-based tests for cointegration, and they all exploit the fact that in the spurious regression case (null hypothesis of non-cointegration) the residuals are not $I(0)$: A DF test, for example, would examine the statistical significance of the t ratio on $\rho - 1$ in the regression:

$$\Delta\hat{\varepsilon}_t = a_0 + (\rho - 1)\hat{\varepsilon}_{t-1} + u_t;$$

where u_t is supposed to be an iid sequence. The evidence against the null of non-cointegration increases as this t ratio becomes more negative. The critical values to be used, are not however those of the DF distribution, since the DF test is applied on the estimated residuals, $\hat{\varepsilon}_t$, and not on ε_t.

Unfortunately, the standard DF test has other drawbacks. One of them is that its power depends critically on the value of the AR parameter ρ (< 1) and is usually very low for values of ρ close to unity. The DF test also relies on the assumption that the variable follows an AR(1) process with iid disturbances $\{u_t\}_{t \geq 1}$, which is rarely met in practice in economic time series, since these disturbances are usually correlated. A well-known solution to this serial correlation problem is to run an ADF test on the series. This modification is flexible enough to account for the serial dependence in the disturbances by entering lagged values of the dependent variable in the regression

$$\Delta\varepsilon_t = a_0 + (\rho - 1)\varepsilon_{t-1} + \sum_{i=1}^{p} a_i \Delta\varepsilon_{t-i} + w_t,$$

where p is chosen so as to ensure that the residuals w_t are white noise. Another device that accounts for serial correlation is the nonparametric correction to the standard DF test, referred to as the PP test, after Phillips and Perron (1988), which eliminate the nuisance parameters present in the DF test statistic when the disturbances $\{u_t\}_{t \geq 1}$ are not an iid sequence.

Residual-based cointegration tests have, in general, low power since they focus on the error dynamics ignoring the static equation dynamics. The existence of an

Error Correction Model (ECM) representation for any cointegrating relationship, as shown by Engle and Granger (1987), suggests an alternative class of cointegration tests with improved power performances. These tests consist of two stages. In the first one, the cointegrating parameter α is "superconsistently" estimated by an OLS regression. In the second, the cointegrating residuals, $\varepsilon_t = y_t - \alpha x_t$, are plugged in the short-run dynamics of y_t, as follows:

$$\Delta y_t = \Theta(B)\Delta x_t + \Phi(B)\Delta y_{t-1} - \lambda(y_{t-1} - \alpha x_{t-1}),$$

where $\Theta(B)$ and $\Phi(B)$ are polynomials in the delay operator B. The previous equation represents the ECM in its basic form, and suggests the idea that changes in the variables are constrained by the final objective of reaching a target equilibrium, namely $y_t = \alpha x_t$. An alternative interpretation considers this equilibrium error as the result of agents' forecasts of these changes (Campbell and Shiller, 1988). Testing for $\lambda = 0$ in the ECM equation amounts at testing the null hypothesis of no cointegration, with the advantage that such test now takes into account the contemporaneous effect of Δx_t on Δy_t.

Today, cointegration tests are widely used in the econometric practice. They suggest restrictions to be imposed on multivariate or vector autoregressive (VAR) models and can be used to test economic theories such as the market efficiency hypothesis (the hypothesis that the prices in two different markets have a long-run equilibrium), or the purchasing power parity (establishing that the exchange rate between two countries are proportional to the ratio of their price levels).

Cointegration analysis also provides answers to a number of related questions of practical relevance, such as how fast and in which way (linear or nonlinear) arbitrage removes price differences, and for which commodities (see Maddala and Moo-Kim, 1998, for details).

1.2 Departures from the standard assumptions

Economic variables are often transformed, usually by taking logarithms or other monotonic transformations (i.e. Box-Cox-type transformations) before the modeling and analysis stages. On the other hand, the economic theory often suggests a nonlinear relationship for two or more variables. As shown by Granger and Hallman (1988,1991a) and by Ermini and Granger (1993), many transformations of an integrated time series still yield time series with similar long-run properties. Thus the concept of cointegration could be extended in the following sense (see Granger and Terasvirta, 1993, for a definition of nonlinear cointegrating relationships): two $I(d)$ $(d > 0)$ univariate time series x_t and y_t are nonlinearly cointegrated if there exist a couple of functions $f(.)$ and $g(.)$ so that $f(x_t)$ and $g(y_t)$ are integrated, or $I(d')$ $(d' > 0)$; but $f(x_t) - g(y_t)$ is $I(0)$. If $f(.)$ is invertible the previous condition amounts at finding a function $h = f^{-1} \circ g$ so that $x_t - h(y_t)$ is $I(0)$.

In practice, finding the appropriate transformations is often an impossible task, thus the interest focusses on the construction of cointegration testing devices that are invariant to transformations of the series, or at least to transformations within a given class (i.e. the class of monotonic transformations). Motivated by the bad performance of the DF unit root test on monotonically transformed $I(1)$ time series, Granger and Hallman (1991b) proposed using the ADF test on the ranks series from the original variables and their regression residuals (the so-called RADF test) so as to obtain a unit root test invariant to monotonic non-linear transformations. Breitung and Gouriéroux (1997) derived the asymptotic properties of the RADF test. The former also proposed a rank test for cointegration (Breitung, 1998) that explotes the intuition that the sequences of ranks diverge under non-cointegration and evolve similarly otherwise. This test seems to outperform its parametric counterparts when the true relationship is nonlinear and monotonic. However, it su¤ers from size bias when the series have short-run correlations and the null hypothesis required independent random walks. In the same line of research, Aparicio and Escribano (1998) proposed a few test statistics based on the mutual information in an attempt to provide a nonparametric characterisation of strong serial dependence, on the one hand, and a device which detects nonlinearities in cointegrating relationships, on the other.

Another problem which can alter dramatically the outcome of cointegration analysis is the omission of relevant variables in the model to account, for example, for the presence of breaks. Breaks a¤ect both unit root and cointegration tests. Standard unit root tests tend to be “over-conservative” of the null hypothesis on time series with breaks (Perron, 1990) and the bias increases with increasing break magnitude (Montañés and Reyes, 2000). In addition, certain variables co-evolve in the long-run only when other explanatory variables are taken into account. This could explain why, for example, the US wages and prices do not seem to be cointegrated, as remarked by Engle and Granger (1987). Finally, as shown by Malliaropulos (2000) when analysing the relationship between in‡ation and nominal interest rates (the so-called "Fisher e¤ect"), it is also possible that cointegration be just an artifact of two $I(0)$ series being a¤ected by a common break. Most cointegration devices dealing with level breaks focus on, …rst, testing for the presence of structural changes at a given time instant, and second, …ltering them out after testing for their number and their locations (see Maddala and Moo Kim, 1998, for a nice review of the major contributions).

Besides structural breaks and neglected nonlinearities, outliers or atypical observations may also have deleterious e¤ects on unit root and cointegration tests. As shown by Franses and Haldrup (1994), outliers induce a negative moving average component in the model errors of a unit root time series. As a consequence, standard unit root tests may exhibit important size distortions and a tendency towards spuriously rejecting the null hypothesis. Robust unit root testing procedures against outlying observations have been suggested by Stock (1999) and Vogelsang (1999), but these suggestions have not been analysed in the context of cointegration testing.

The possibility of structural breaks and outliers in real time series have possibly led to an overuse of proxy and dummy variables in the modeling practice, at the risk of explaining what is in reality a cointegrating relationship in a small sample of data. An alternative avenue of research in cointegration analysis is semiparametric and nonparametric testing. In this paper, we propose a nonparametric cointegration test which exploits the statistical properties of the records from a time series, and inherit a number of desirable properties from them. is invariant to monotonic nonlinearities and robust to the presence of level breaks and additive outliers.

The structure of the paper is as follows. In Section 1 we introduce a number of test statistics for the purpose of testing cointegration. These statistics are related to record counting processes. The asymptotic distribution under the null of independent random walks is obtained for the test statistic based on the standardized number of co-records from the pair of time series variables. This null distribution is also useful when stationary forms of weak dependence are allowed in the time series variables, either because of the statistical properties of records or because a prewhitening …lter has been previously applied on the regression residuals. We also discuss the null distribution of another record-based test statistic, which tests for unit roots on these residuals, an that we call the single record-counting cointegration (SRCC) test. The latter inherits the properties of the Range Unit Root (RUR) test of Aparicio, Escribano and Garcia (2003a). The behavior of the CRCC test is presented in Section 2. In particular, we analyse its consistency, its invariance with respect to monomotinic nonlinearities, and its robustness to level breaks. However, the CRCC test is affected by the presence of early additive ouliers. Thus a modification of the CRCC test is introduced and analysed in Section 3 to cope succesfully with this problem. After the concluding remarks in Section 4, an Appendix is devoted to proving the main results.

2 RECORD-BASED COINTEGRATION TEST STATISTICS

In this section, we discuss a few alternative ways of using record counting statistics for the purpose of testing cointegration in a pair of time series variables. All these statistics are related to the total number of records in a sample of size n. This quantity can be represented by $\sum_{i=1}^{n} 1(\Delta R_i^{(x)} > 0)$, where $1(.)$ denotes the indicator function and $\{R_i^{(x)}\}_{i\geq 1}$ represents the range sequence for x_t.

Range statistics are well-known in the analysis of the distributions of partial sums and empirical process (see for instance, Shorack and Wellner, 1986). The range of a data sample is defined in terms of its extremes. Formally, for a given time series x_t, the statistics $x_{1,i} = \min\{x_1, \cdots, x_i\}$ and $x_{i,i} = \max\{x_1, \cdots, x_i\}$ are called the i-th extremes. When the sample comes from a time series x_t, a monotonically increasing sequence of ranges can be obtained as $R_i^{(x)} = x_{i,i} - x_{1,i}$, for $i = 1, 2, 3, \cdots, n$, where n denotes the sample size.

Aparicio, Escribano and Garcia (2000) proposed a pair of complementary nonparametric test statistics, $\rho_{x,y}^{(n)}$ and $\eta_{x,y}^{(n)}$, for testing cointegration using the range sequences of the variables. These statistics were defined as follows:

$$\rho_{x,y}^{(n)} = \frac{\sum_{i=2}^{n} \Delta R_i^{(x)} \Delta R_i^{(y)}}{\left(\sum_{i=2}^{n} \left[\Delta R_i^{(x)}\right]^2\right)^{1/2} \left(\sum_{i=2}^{n} \left[\Delta R_i^{(y)}\right]^2\right)^{1/2}} \qquad (1)$$

$$\kappa_{x,y}^{(n)} = \frac{\sum_{i=1}^{n} \left[1(\Delta R_i^{(x)} > 0)1(\Delta R_i^{(y)} = 0) + 1(\Delta R_i^{(x)} = 0)1(\Delta R_i^{(y)} > 0)\right]}{\sum_{i=1}^{n} 1(\Delta R_i^{(x)} = 0)1(\Delta R_i^{(y)} = 0)}. \qquad (2)$$

By means of Monte Carlo simulations, these authors showed the possibility of combining the outcomes of the corresponding tests to discriminate between linear cointegration, monotonic nonlinear cointegration, independent random walks, and comoving or short-run dependent random walks in finite samples. However, nothing was said about the asymptotic behavior of such tests.

Another pair of record statistics were later proposed by the same authors (Aparicio, Escribano and Garcia, 2003a,b) for robust unit root testing. The key idea relied on the different vanishing rates of the long-run frequency of a new record, $n^{-1} \sum_{t=1}^{n} 1(\Delta R_t^{(x)} > 0)$, for an $I(1)$ and an $I(0)$ time series, in such a way that the normalized long-run frequency of records

$$J_x^{(n)} = n^{-1/2} \sum_{t=1}^{n} 1(\Delta R_t^{(x)} > 0) \qquad (3)$$

converged in probability to zero under the alternative of stationarity, and to a nondegenerate positive random variable under the null hypothesis of a unit root.

In this paper, we discuss and analyse the properties of two alternative robust cointegration testing devices involving the records of the time series. One of such devices is based on the evaluation of the test statistic $J_x^{(n)}$ on the residuals from the regression of y_t on x_t, while the other is closely connected with the joint number of records (hereafter, co-records) of the series, that is with:

$$T_{x,y}^{(n)} = \sum_{t=1}^{n} 1(\Delta R_t^{(x)} > 0)1(\Delta R_t^{(y)} > 0). \qquad (4)$$

2.1 The asymptotic distribution of the number of co-records under the null of independent random walks

Here we establish the asymptotic distribution of the standardized test statistic based on the number of co-records, $T_{x,y}^{(n)}$ (hereafter Co-Record Counting Cointegration -CRCC- test statistic); under the null of two independent random walks. Then we will show that this result can be exploited for testing the null hypothesis of non-cointegration.

Theorem 1 *Let the processes $x_t = \sum_{i=1}^{t} \varepsilon_i$; $y_t = \sum_{i=1}^{t} \xi_i$ for $t = 1, 2, ..., \infty$; where ε_i and ξ_i are independent continuous zero-mean iid sequences with finite variances σ_ε^2 y σ_ξ^2, respectively, and symmetric pdf's. Let $T_{x,y}^{(n)}$ be the number of joint records of x_t and y_t in a sample of size n, that is:*

$$T_{x,y}^{(n)} = \sum_{t=1}^{n} 1(\Delta R_t^{(x)} > 0)1(\Delta R_t^{(y)} > 0), \qquad (5)$$

then

$$(\log n)^{-1} T_{x,y}^{(n)} \xrightarrow{p} 1 \qquad (6)$$

and

$$\lim_{n \to \infty} P\left\{ (\sigma^2 \log n)^{-1/2} \left[T^{(n)}_{x;y} - \mu \log n \right] > z \right\} = 1 - \Phi(z) \tag{7}$$

for any positive real number z; and for two constants μ and σ which can be consistently estimated as:

$$\hat{\mu}_n = \lim_{m \to \infty} \frac{1}{m \log n} \sum_{i=1}^{M} T^{(n)(i)}_{x;y} \tag{8}$$

$$\hat{\sigma}^2_n = \lim_{m \to \infty} \frac{1}{m \log n} \sum_{i=1}^{M} \left(T^{(n)(i)}_{x;y} - \hat{\mu}_n \log n \right)^2; \tag{9}$$

with $T^{(n)(i)}_{x;y}$ representing the number of co-records for the i-th pair of independent random walks x_t and y_t with sample size n:

Proof. See Appendix A1. ■

It is surprising that an identical limit distribution and scaling behavior for both the mean and the variance is exhibited by the standardized record counting process of an iid sequences, as shown in Embrechts, Kluppelberg and Mikosch (1997, p. 257). This suggests that record counting process have similar asymptotic properties for iid sequences; for stationary time series, and even for heterogeneous but weakly-dependent time series as the first differences of the range sequences from $I(1)$ time series.

Indeed, a well-known result from extremal theory is that the statistical properties of records from iid sequences of random variables are shared by a wide class of dependent stationary time series (see for instance, Lindgren and Rootzén, 1987, and Leadbetter and Rootzén, 1988). This prompts the question of whether short-run dependencies and cross-dependencies may have an impact or not on a record-based test for cointegration. Thus consider, for example, as our null hypothesis the hypothesis of non-cointegration in its simpler form, which can be expressed as:

$$H_0 : \Delta y_t = a \Delta x_t + w_{1t}; \ \ \Delta x_t = w_{2t}; \tag{10}$$

where $\{w_{1t}\}_t$ and $\{w_{2t}\}_t$ are independent sequences of iid zero-mean random variables, and a is the short-run correlation parameter. Letting

$$w_t = \sum_{i=1}^{t} w_{1i}; \tag{11}$$

it can be seen that, for any $a \neq 0$; we have:

$$\begin{aligned} \sum_{t=1}^{n} & 1(\Delta R_t^{(x)} > 0) 1(\Delta R_t^{(y)} > 0) \\ &= \sum_{t=1}^{n} 1(\Delta R_t^{(ax)} > 0) 1(\Delta R_t^{(y)} > 0) \\ &= \sum_{t=1}^{n} 1(\Delta R_t^{(y-w)} > 0) 1(\Delta R_t^{(y)} > 0): \end{aligned} \tag{12}$$

Therefore the limit behavior of $(\log n)^{-1/2} T_{x;y}^{(n)}$ does not depend on a as long as $a \neq 0$. This is an important result since the asymptotic null distribution of standard cointegration tests is affected by short-run dependencies.

2.2 Alternative record-based cointegration testing schemes

In this section we propose three alternative testing strategies based on record counting statistics. Consider again the null hypothesis H_0 of non-cointegration of the previous section, and suppose we have a consistent estimator of the parameter a which describes the short-run relationship between the random walks x_t and y_t. Let $\hat{a}_n$ represent such estimator for a sample size n and define the short-run regression residuals as $\hat{w}_t = \Delta y_t - \hat{a}_n \Delta x_t$. Now $\tilde{w}_t = \sum_{i=1}^{t} \hat{w}_{1i}$ will be an $I(1)$ process uncorrelated in the short-run with x_t, that is $E\{\Delta \tilde{w}_t \Delta x_t\} = 0$. Under the null of non-cointegration, and assuming Gaussianity for the innovations w_{1t} and w_{2t}, $\tilde{w}_t$ will be independent of x_t. Therefore the conditions of Theorem 1 hold for the pair of series $\{\tilde{w}_t, x_t\}$, and we could find two non-negative real numbers σ and μ such that:

$$(\sigma^2 \log n)^{-1/2} \left(T^{(n)}_{\tilde{w},x} - \mu \log n \right) \Rightarrow N(0,1) \quad \text{under } H_0 \tag{13}$$

Under the alternative hypothesis H_1 of cointegration, it will be shown in the next section that the CRCC test statistic

$$(\log n)^{-1/2} \left(T^{(n)}_{\tilde{w},x} - \mu \log n \right)$$

diverges to infinity.

Another possibility consists in applying the RUR test of Aparicio, Escribano and Garcia (2003a) on the residuals $\hat{\xi}_t$ estimated from the long-run relation $\tilde{w}_t = \alpha x_t + \xi_t$, that is $\hat{\xi}_t = \tilde{w}_t - \hat{\alpha}_n x_t$, where $\hat{\alpha}_n$ could be the OLS estimate of α based on the sample of size n. We call this device the Single-Record Counting Cointegration (SRCC) test. An improved version of the RUR test known as the Forward-Backward RUR (FB-RUR) test (see Aparicio, Escribano and Garcia, 2003b) could also be used for this purpose. The FB-RUR test was conceived to prevent size distortions from eventual early additive outliers in the series.

In the followin, we use $\hat{\xi}_t$ as the dependent variable instead of y_t in order to avoid the small-sample biases caused by unremoved short-run dependencies between the series. For convenience, we recall the expressions of both range-based unit root test statistics when applied on these residuals:

$$J_{\hat{\xi}}^{(n)} = n^{-1/2} \sum_{t=1} 1(\Delta R_t^{(\hat{\xi})} > 0), \text{ for the RUR test,} \tag{14}$$

$$J_{\hat{\xi}}^{(n)(*)} = (2n)^{-1/2} \sum_{t=1} \left\{ 1(\Delta R_t^{(\hat{\xi})} > 0) + 1(\Delta R_t^{(\hat{\xi}')} > 0) \right\}; \text{ for the FB-RUR test,} \tag{15}$$

where $\hat{\xi}'_t = \hat{\xi}_{n-t+1}$.

Under the null of non-cointegration, $\hat{\xi}_t \sim I(1)$; and according to lemma 2:

$$P\{J_{\hat{\xi}}^{(\infty)} < h\} = \frac{1}{2\sqrt{2\pi}} \int_0^h \exp\left(-\frac{v^2+2}{4}\right)[1 - \phi(v)]dv; \tag{16}$$

whereas under the alternative of cointegration, $\hat{\xi}_t \sim I(0)$; which gives $J_{\hat{\xi}}^{(n)} \xrightarrow{p} 0$.

Notice that this testing device does not require the Gaussianity of the innovations w_{1t} and w_{2t}; and since $P(J^{(\infty)} = 0 | x_t; y_t$ independent random walks$) = 0$, the test is consistent.

Finally, a third testing device exploits the range properties of the integrated long-run residuals $\hat{S}_t = \sum_{i=1}^{t} \hat{\xi}_i$ by means of the record test statistic $T_{\cdot;x}^{(n)}$: Under the null of non-cointegration, H_0;we have $\hat{S}_t \sim I(2)$. Consequently, under

H_0 $(\log n)^{-1/2}\left(T^{(n)}_{\hat{\varepsilon},x} - \mu \log n\right) \sim (\log n)^{-1/2} n^{1/2}$ (see Appendix A2 for details) which diverges as n grows to infinity. We should then expect rejection of the null. Under the alternative of cointegration $\hat{\varepsilon}_t \sim I(1)$; which will also be independent of x_t if we assume the Gaussianity of the innovations w_{1t} and w_{2t}. Inverting the hypotheses for convenience, under the null H_0' of cointegration, $(\log n)^{-1/2}\left(T^{(n)}_{\hat{\varepsilon},x} - \mu \log n\right) = O(1)$ and will converge to a non-degenerate random variable whose distribution is given in the theorem. Under the alternative H_1' of cointegration $(\log n)^{-1/2}\left(T^{(n)}_{\hat{\varepsilon},x} - \mu \log n\right)$ will diverge. Such a procedure is equivalent to the evaluation of $(\log n)^{-1/2}\left(T^{(n)}_{\hat{w},x} - \mu \log n\right)$; since in both cases the test statistic diverges under the alternative at the same rate of $(\log n)^{-1/2} n^{1/2}$ (see Appendix A2).

Cointegration tests based on either RUR test statistics inherit the robustness and invariance properties of the latter in the face of certain deviations from the standard assumptions. Thus for instance, these tests will be invariant with respect to monotonic transformations of the individual series and with respect to the innovations variances. Besides, they will be robust in the presence of level breaks or additive outliers. In the sequel, it is shown that these properties are also shared by the CRCC test.

3 PROPERTIES OF THE CRCC TEST

3.1 Consistency

If x_t and y_t are cointegrated then for some $a \neq 0$ there exists an $I(0)$ sequence, η_t; such that $y_t = ax_t + \eta_t$:

Since for large t x_t will dominate η_t, we can write:

$$T_{x;y}^{(n)} = \sum_{t=1} 1(\Delta R_t^{(x)} > 0)1(\Delta R_t^{(y)} > 0) \simeq \sum_{t=1} 1(\Delta R_t^{(x)} > 0): \tag{17}$$

But from lemma 2:

$$\sum_{t=1} 1(\Delta R_t^{(x)} > 0) = J_x^{(n)} = O(n^{1/2}): \tag{18}$$

Thus under the alternative hypothesis of cointegration, the normalized test statistic will satisfy:

$$(\log n)^{-1/2}\left(T_{x;y}^{(n)} - \mu \log n\right) \xrightarrow{p} \infty: \tag{19}$$

Now suppose $z_t \sim I(0)$ and independent of x_t: It is shown in Appendix 2 that for any $\mu > 0$:

$$(\log n)^{-1/2}(T_{z;x}^{(n)} - \mu \log n) \xrightarrow{p} -\infty: \tag{20}$$

Therefore the standardized number of co-records allows the discrimination between pairs of independent or comoving random walks, pairs of cointegrated variables, and pairs of stationary time series and random walks.

3.2 Invariance against Monotonic Nonlinearities

Monotonic transformations preserve the ordering of the observations in any time series, and thus the inter-record times. As a consequence, if we let $f(\cdot)$ and $g(\cdot)$ be monotonic nonlinear transformations, we must have:

$$T^{(n)}_{f(x);g(y)} = T^{(n)}_{x;y}. \tag{21}$$

More generally, let x_t and y_t be $I(1)$ time series variables, and let $x'_t = f(x_t) + \varepsilon_t;\ y'_t = g(y_t) + \eta_t$, where $\{\varepsilon_t\}_{t \geq 1};\ \{\eta_t\}_{t \geq 1}$ are independent iid sequences with zero-mean and finite variances. Since for large $t;\ f(x_t)$ and $g(y_t)$ will dominate ε_t and η_t respectively, the records of x'_t (y'_t) will occur at almost the same instants as those of x_t (y_t). As a consequence, the co-record counts will tend to be the same for both pairs of series. That is:

$$\begin{aligned} T^{(n)}_{x';y'} &= \sum_{t=1} 1(\Delta R^{(x')}_t > 0)1(\Delta R^{(y')}_t > 0) \\ &\simeq T^{(n)}_{x;y} = \sum_{t=1} 1(\Delta R^{(x)}_t > 0)1(\Delta R^{(y)}_t > 0). \end{aligned} \tag{22}$$

In finite samples, the actual size will oscillate around the nominal size depending on the type of transformations. For example, certain classes of transformations can emphasize the $I(1)$ part over the $I(0)$ part. This feature may lead, in finite samples, to size fluctuations around the nominal one.

3.3 Robustness against level breaks

Unit root and cointegration tests implicitely assume that the deterministic trend is properly specified. This need not be the case in practice, especially when dealing with long series. Policy changes, economic depressions, price shocks entail parameter changes in a time series model. These parameter changes are usually referred to as structural breaks. Several authors (see for instance Rappoport and Reichelin,1989; Perron 1989; Hendry and Neale, 1991; Gregory, Nason and Watt, 1996; Campos, Ericsson and Hendry, 1996, to name a few) have reported on a tendency of standard unit root tests to underreject the null of a unit root in the presence of a break. On the other hand, unaccounted breaks can change dramatically the outcome of a cointegration test. For example, Muscatelli and Papi (1990) find no evidence of a cointegrating relationship among the variables "money", "prices", "income" and "interest rate" unless a dummy variable for financial innovation in the 70's and 80's is included in the model. Other authors have reported similar findings in modeling other long-run relationships (see for instance, Drobny and Hall, 1989; Hall et al., 1989; Muscatelli et al., 1990). Also, the omission of variables is also the reason why no empirical evidence of cointegration was found between US wages and prices, or US money and prices (Engle and Granger, 1987). This problem has prompted researchers to increasingly use dummy variables as a way of explaining structural changes and preventing the latter from inducing an spurious long-run relationship.

The trouble with the previous approach is that a set of dummies can always be found to account for the eventual $I(1)$ nature of the regression residuals, thereby biasing a cointegration test towards rejecting the null of non-cointegration. An alternative approach to the use of dummy variables consists in allowing for a time-varying cointegration parameter, as suggested by Granger (1986). But time-varying relationships are difficult to estimate on small data samples as those frequently encountered in macroeconomics.

Our approach bypasses the previous difficulties by proposing a cointegration test robust to the presence of structural breaks in the series. In this way, there is no need to explicitedly account for such changes in the model.

Define the processes:

$$x_t = \rho_1 x_{t-1} + \varepsilon_t; \tag{23}$$

$$z_t = \rho_1 z_{t-1} + \varepsilon_t + \sum_{i=1}^{m} s_i 1(t = t_i); \tag{24}$$

$$y_t = \rho_2 y_{t-1} + \xi_t; \tag{25}$$

where $|\rho_i| < 1$ and $\varepsilon_t; \xi_t$ representing independent sequences of zero-mean iid random variables. Notice that z_t is the same process as x_t except for the presence of m level breaks of magnitud s_i at instants $t_i \in [1; n]$: Suppose without loss of generality that $s_i >> 0; \forall i$, so that $\Delta R^{(z)}_{t_i} = s_i$ with probability 1: Since the instants $\{t_i\}_{i=1;m}$ form a set of Lebesgue measure zero, we have with probability one that $(\log n)^{-1=2}(T^{(n)}_{x;y} - T^{(n)}_{z;y}) = 0$ and for any value of m as long as $m = o(\log n)$:

Indeed,

$$
\begin{aligned}
T_{z,y}^{(n)} &= \sum_{\substack{t=1 \\ t\neq t_i}}^{n} 1(\Delta R_t^{(z)} > 0)1(\Delta R_t^{(y)} > 0) + \sum_{i=1}^{m} 1(\Delta R_{t_i}^{(z)} > 0)1(\Delta R_{t_i}^{(y)} > 0) \\
&= \sum_{\substack{t=1 \\ t\neq t_i}}^{n} 1(\Delta R_t^{(z)} > 0)1(\Delta R_t^{(y)} > 0) + \sum_{i=1}^{m} 1(\Delta R_{t_i}^{(y)} > 0) \\
&\leq \sum_{\substack{t=1 \\ t\neq t_i}}^{n} 1(\Delta R_t^{(z)} > 0)1(\Delta R_t^{(y)} > 0) + m \qquad (26) \\
&\leq \sum_{t=1}^{n} 1(\Delta R_t^{(x)} > 0)1(\Delta R_t^{(y)} > 0) + m \\
&\leq T_{x,y}^{(n)} + m.
\end{aligned}
$$

Therefore

$$
(\log n)^{-1/2}(T_{z,y}^{(n)} - T_{x,y}^{(n)}) \leq m(\log n)^{-1/2} \to 0, \qquad (27)
$$

as $n \to \infty$ and as long as $m = o(\log n)^{1/2}$.

3.4 Robustness against Additive Outliers

Let $u_t = u_{t-1} + \varepsilon_t$, $y_t = y_{t-1} + \xi_t$, with ε_t, ξ_t representing independent sequences of zero-mean iid random variables. Suppose u_t is contaminated by Additive Oultiers (AO's). For simplicity, we may restrict our analysis to the case of a single AO of magnitude $s > 0$ ocurring at time $t = t_1$. The contaminated series, x_t, can be written as:

$$
x_t = u_t + s\,1(t = t_1). \qquad (28)
$$

Notice that

$$\Delta R_{t_1}^{(x)} = s + \Delta R_{t_1}^{(u)} > 0 \tag{29}$$

and that for s large enough

$$\Delta R_t^{(x)} = 0 \quad \forall t > t_1; \tag{30}$$

with probability one. In such case

$$T_{x,y}^{(n)} = \sum_{t=1}^{n} 1(\Delta R_t^{(x)} > 0)1(\Delta R_t^{(y)} > 0) = T_{u,y}^{(t_1)}. \tag{31}$$

Now since

$$(\log n)^{-1/2} T_{x,y}^{(n)} = (\log n)^{-1/2} T_{u,y}^{(t_1)} \approx (\log n)^{-1/2} \log t_1; \tag{32}$$

for any positive real numbers σ and μ:

$$(\sigma^2 \log n)^{-1/2} \left(T_{x,y}^{(n)} - \mu \log n \right) \xrightarrow{p} -\infty, \text{ as } n \to \infty \text{ when } t_1 = o(n). \tag{33}$$

Thus the actual size of the test, given by $P\left\{ (\sigma^2 \log n)^{-1/2} \left(T_{x,y}^{(n)} - \mu \log n \right) > t_{\alpha,n} | H_0 \right\}$ with $t_{\alpha,n}$ denoting the critical value of the null distribution of $(\sigma^2 \log n)^{-1/2} \left(T_{x,y}^{(n)} - \mu \log n \right)$ at the α significance level, will tend to be below the nominal size, α, and will approach zero as n grows to infinity:

$$P\left\{ (\sigma^2 \log n)^{-1/2} \left(T_{x,y}^{(n)} - \mu \log n \right) > t_{\alpha,n} | H_0 \right\} \to 0; \quad \text{as} \quad n \to \infty. \tag{34}$$

4 THE FORWARD-BACKWARD RECORD COUNTING COINTEGRATION (FB-CRCC) TEST

A Forward-Backward Record-Counting Cointegration (FB-CRCC) test, similar in spirit to the FB-RUR test proposed in Aparicio, Escribano and Garcia (2003b), can be used to cope with the size distortion problem caused by early large outliers in the series. Consider test statistic:

$$T_{x,y}^{(n)(*)} = T_{x,y}^{(n)} + T_{x',y'}^{(n)}, \tag{35}$$

where x_t' and y_t' are the specular images of x_t and y_t, that is:

$$x_t' = x_{n-t+1}, \quad t = 1, \ldots, n \tag{36}$$

$$y_t' = y_{n-t+1}, \quad t = 1, \ldots, n$$

Since the effective sample size is now twice the original the appropriate scaling for $T_{x,y}^{(n)(*)}$ would be $(\log 2n)^{-1}$. However notice that

$$\begin{aligned}
&(\log 2n)^{-1/2} T_{x,y}^{(n)(*)} \\
&= \frac{(\log n)^{-1/2}}{\left(1 + \frac{\log 2}{\log n}\right)^{1/2}} \sum_{t=1}^{n} \left\{ 1(\Delta R_t^{(x)} > 0) 1(\Delta R_t^{(y)} > 0) + 1(\Delta R_t^{(x')} > 0) 1(\Delta R_t^{(y')} > 0) \right\}
\end{aligned} \tag{37}$$

$$\simeq (\log n)^{-1/2} \left\{ T_{x,y}^{(n)} + T_{x',y'}^{(n)} \right\}, \text{ for large enough } n.$$

When x_t is the contaminated $I(1)$ series of the previous section, given by $x_t = u_t + s\,1(t = t_1)$, where $s >> 0$, then for large n:

$$(\log 2n)^{-1/2} T_{x,y}^{(n)(*)} \simeq (\log n)^{-1/2} T_{u,y}^{(t_1)} + (\log n)^{-1/2} T_{u',y'}^{(n-t_1)}. \tag{38}$$

Thus in the most insidious case of an early outlier at $t_1 = o(n)$, we have for large n:

$$\left(\sigma^2 \log 2n\right)^{-1/2}\left(T^{(n)(*)}_{x;y} - \mu \log 2n\right) \approx \left(\sigma^2 \log n\right)^{-1/2}\left(T^{(n)}_{u';y'} - \mu \log n\right). \tag{39}$$

If the distribution of disturbances ϵ_t and ξ_t are symmetric around their zero mean then the random walks u_t and y_t are reversible and, for appropriate positive values of σ^2 and μ; $\left(\sigma^2 \log 2n\right)^{-1/2}\left(T^{(n)(*)}_{x;y} - \mu \log 2n\right)$ will converge weakly towards a standard Normal random variable, as claimed in Theorem 1.

5 CONCLUDING REMARKS

In this paper, a number of consistent cointegration testing methods based on record counting processes have been proposed. These methods exploit the robustness of record statistics in the face of deviations from the assumptions made by standard cointegration tests. In particular, it is shown that the new statistics are invariant with respect to both monotonic transformations and to the variance of the individual series. The tests are robust to level breaks, and can be made robust to additive outliers by means of a simple device which averages the statistic values for the original and the time-reversed series. A comparison of the performances between this novel methodology and the standard cointegration tests, and its application to real data, will be considered in a second paper. Possible extensions to more general scenarios will also be discussed.

Appendix A1

Lemma 2 Let $x_t = \sum_{i=1}^{t} \varepsilon_i$ where $\{\varepsilon_i\}_{i \geq 1}$:are continuous iid random variables with bounded and symmetric pdf, zero mean and finite variance σ_ε^2. Suppose that x_0 has also a bounded pdf and finite variance. And let $J_x^{(n)} = n^{-1/2} \sum_{t=1}^{n} 1(\Delta R_t^{(x)} > 0)$. Then we have

$$P\left\{J_x^{(\infty)} \leq h\right\} = \frac{1}{2\sqrt{2\pi}} \int_0^h \exp\left(-\frac{v^2+2}{4}\right)[1 - \Phi(v)]dv;$$

where $\Phi(.)$ is the probability distribution function of a standard Normal random variable.

Proof. See Aparicio, Escribano and Garcia (2003a). ∎

Lemma 3 Let the processes $x_t = x_{t-1} + \varepsilon_t$; $z_t = \rho z_{t-1} + \varepsilon_t$; $y_t = y_{t-1} + x_t$ for $t = 1, 2, \ldots, \infty$; where $|\rho| < 1$ and $\{\varepsilon_t\}_{t \geq 1}$ is a sequence of zero-mean and finite variance σ_ε^2:iid random variables. Then for any t:

$$\begin{aligned}
P\left\{\Delta R_{t+j}^{(x)} > 0 | \Delta R_t^{(x)} > 0\right\} &= P\left\{\Delta R_j^{(x)} > 0\right\} \\
\lim_{j \to \infty} P\left\{\Delta R_{t+j}^{(x)} > 0 | \Delta R_t^{(x)} > 0\right\} &= \lim_{j \to \infty} P\left\{\Delta R_{t+j}^{(x)} > 0\right\} = 0 \\
\lim_{j \to \infty} P\left\{\Delta R_{t+j}^{(z)} > 0 | \Delta R_t^{(z)} > 0\right\} &= \lim_{j \to \infty} P\left\{\Delta R_{t+j}^{(z)} > 0\right\} \\
P\left\{\Delta R_{t+j}^{(y)} > 0 | \Delta R_t^{(y)} > 0\right\} &= P\left\{\Delta R_j^{(y)} > 0\right\} = P\left\{\Delta R_{t+j}^{(y)} > 0\right\} = 1; \quad \forall j.
\end{aligned}$$

Proof. Let us denote by $\tau_1^{(x)}$ the first ladder epoch of x_t; that is the first time instant at which a record for x_t occurs, and denote by $\tau_1^{(x)} + \ldots + \tau_k^{(x)}$ the kth ladder index or kth record time of x_t. Following Feller (1971, vol. 2, 1971), the random variables $\left\{\tau_i^{(x)}\right\}_{i \geq 1}$ are iid. We use this property to prove this lemma.

$$
\begin{aligned}
&P\left\{\Delta R_{t+j}^{(x)} > 0 \mid \Delta R_t^{(x)} > 0\right\} \\
&= P\left\{t + \tau_1^{(x)} + \dots + \tau_k^{(x)} = t + j, \text{ for some integer } k \in (0, j] \mid t \text{ is a ladder index}\right\} \\
&= P\left\{\tau_1^{(x)} + \dots + \tau_k^{(x)} = j, \text{ for some integer } k \in (0, j] \mid t \text{ is a ladder index}\right\} \\
&= P\left\{\tau_1^{(x)} + \dots + \tau_k^{(x)} = j, \text{ for some integer } k \in (0, j]\right\} \\
&= P\left\{\Delta R_j^{(x)} > 0\right\}.
\end{aligned}
$$

It can be shown similarly that

$$P\left\{\Delta R_{t+j}^{(y)} > 0 \mid \Delta R_t^{(y)} > 0\right\} = P\left\{\Delta R_j^{(y)} > 0\right\}.$$

Now following lemma 2:

$$P\left\{\Delta R_{t+j}^{(x)} > 0\right\} - P\left\{\Delta R_j^{(x)} > 0\right\} \sim (t + j)^{-1/2} - j^{-1/2} \to 0, \text{ as } j \to \infty.$$

For z_t the amount of serial dependence in the sequence $\left\{\Delta R_t^{(z)}\right\}_{t \geq 1}$ is even smaller, so we may expect a similar result. In fact, if z_t is Gaussian then the statistical properties of records are the same as in the iid case (see Lindgren and Rootzén, 1987, and Leadbetter and Rootzén, 1988), and therefore

$$P\left\{\Delta R_t^{(z)} > 0\right\} = O\{(\ln t)^{-1}\}$$

$$P\left\{\Delta R_{t+j}^{(x)} > 0\right\} - P\left\{\Delta R_j^{(x)} > 0\right\} \sim (\ln[t + j])^{-1} - (\ln t)^{-1} \to 0, \text{ .. as } j \to \infty.$$

Finally when $y_t \sim I(2)$ the independence of the events $\left\{\Delta R^{(y)}_{t+j} > 0\right\}$ and $\left\{\Delta R^{(y)}_{t} > 0\right\}$ follows from (see Feller, 1971):

$$P\left\{\Delta R^{(y)}_{t+j} > 0\right\} = P\left\{\Delta R^{(y)}_{j} > 0\right\} = 1; \quad \forall j.$$

■

Lemma 4 Let B_1 and B_2 two independent random variables. And let A be another random variable independent of B_1 and B_2. Define two new random variables as $B_1^* = AB_1$ and $B_2^* = AB_2$. Then B_1^* and B_2^* are independent.

Proof. We may assume without loss of generality that the variables are discrete. Let a be any value such that $P(A = a) > 0$. Since

$$P(B_1|B_2) = P(B_1)$$

we could also write for any such scalar a:

$$P(aB_1|aB_2) = P(aB_1).$$

And also:

$$P(aB_1|aB_2)P(A = a) = P(aB_1)P(A = a);$$

Therefore

$$\begin{aligned} P(AB_1|AB_2) &= \sum_a P(AB_1|AB_2; A = a)P(A = a) \\ &= \sum_a P(AB_1|A = a)P(A = a) = P(AB_1). \end{aligned}$$

■

Lemma 5 Let $\{\xi_i\}_{i\geq 1}$ a sequence of random variables such that $\lim_{i\to\infty} E(\xi_i) = \mu$; and $\lim_{i\to\infty} Var(\xi_i) = 0$. Then

$$\xi_i \xrightarrow{p} \mu.$$

Proof. See Arnold (1990). ■

Lemma 6 Let $\{Z_{n,i};\ i = 1, \ldots, r_n\}$ denote a zero-mean stochastic array, where r_n is a positive, increasing integer-valued function of n, and let

$$T_{r_n} = \prod_{i=1}^{r_n} (1 + i\lambda Z_{n,i}); \quad \text{with } \lambda > 0.$$

Then

$$S_{r_n} = \sum_{i=1}^{r_n} Z_{n,i} \Rightarrow N(0, 1),$$

if the following conditions hold: (a) T_{r_n} is uniformly integrable, (b) $E(T_{r_n}) \to 1$ as $n \to \infty$; (c) $\sum_{i=1}^{r_n} Z_{n,,i}^2 \xrightarrow{p} 1$ as $n \to \infty$; and (d) $\max_{1\leq i\leq r_n} |Z_{n,i}| \xrightarrow{p} 0$ as $n \to \infty$.

Proof. See Davidson (1994, pp. 380-81) ■

Lemma 7 From the stochastic array $\{Z_{n,i}\}$ defined in lemma 5, define

$$\tilde{Z}_{n,i} = Z_{n,i} 1\left(\sum_{j=1}^{i-1} Z_{n,k}^2 \leq 2\right);$$

and let $\tilde{T}_{r_n} = \prod_{i=1}^{r_n}(1 + i\lambda \tilde{Z}_{n,i})$; where $\lambda > 0$. Then $\tilde{T}_{r_n}$ is uniformly integrable if $\sup_n E\left(\max_{1\leq i\leq r_n} Z_{n,i}^2\right) < \infty$. Moreover, if $\sum_{i=1}^{r_n} Z_{n,i}^2 \xrightarrow{p} 1$ then: (a) $\sum_{i=1}^{r_n} \tilde{Z}_{n,i}^2 \xrightarrow{p} 1$ and (b) $\tilde{S}_{r_n} = \sum_{i=1}^{r_n} \tilde{Z}_{n,i}$ has the same limiting distribution as S_{r_n}.

Proof. See Davidson (1994, pp. 382-83). ■

PROOF OF THEOREM 1. Since x_t is a random walk we have from lemma 2:

$$\sum_{t=1} P(\Delta R_t^{(x)} > 0) = O(n^{1/2})$$

$$\begin{aligned}
&\Rightarrow\ P(\Delta R_t^{(x)} > 0) = O(t^{-1/2}) \\
&\Rightarrow\ \left[P(\Delta R_t^{(x)} > 0)\right]^2 = O(t^{-1}) \\
&\Rightarrow\ \sum_{t=1} \left[P(\Delta R_t^{(x)} > 0)\right]^2 = O(\log n);
\end{aligned}$$

since from Euler's formula (see Abramowitz and Stegun, 1972) we can write

$$\sum_{t=1} t^{-1} = \log n + \gamma + \frac{1}{2n} + \frac{1}{12n^2} + O(n^{-4})$$

with $\gamma = 0.57721566$ (Euler's constant).

Now if x_t and y_t are independent we have:

$$\begin{aligned}
E\left\{T_{x,y}^{(n)}\right\} &= \sum_{t=1} P(\Delta R_t^{(x)} > 0) P(\Delta R_t^{(y)} > 0) \\
&= \sum_{t=1} \left[P(\Delta R_t^{(x)} > 0)\right]^2 \\
&= O(\log n).
\end{aligned}$$

Therefore, under H_0, we can write for some positive constant μ:

$$T_{x,y}^{(n)} = \mu \log n + \delta_n V;$$

where V denotes a non-degenerate random variable with unit variance and δ_n defines the asymptotic order for the standard deviation of $T^{(n)}_{x;y}$. Our next objective is to determine δ_n. To do this, first note that

$$E\left(T^{(n)}_{x;y} - \mu\log n\right)^2 = E\left\{\left[T^{(n)}_{x;y}\right]^2\right\} - \mu^2(\log n)^2 = \delta_n^2 E(V^2).$$

$$\begin{aligned}
E\left\{\left[T^{(n)}_{x;y}\right]^2\right\} &= E\left\{\sum_{t=1}^{n}\sum_{t'=1}^{n} 1(\Delta R^{(x)}_t > 0)1(\Delta R^{(y)}_t > 0)1(\Delta R^{(x)}_{t'} > 0)1(\Delta R^{(y)}_{t'} > 0)\right\} \\
&= \sum_{t=1}^{n}\left[P(\Delta R^{(x)}_t > 0)\right]^2 + 2\sum_{t=1}^{n-1}\sum_{t'=t+1}^{n}\left[P(\Delta R^{(x)}_t \Delta R^{(x)}_{t'} > 0)\right]^2 \\
&= \sum_{t=1}^{n}\left[P(\Delta R^{(x)}_t > 0)\right]^2 + W^{(n)}_{x;y} \\
&= \mu\sum_{t=1}^{n} t^{-1} + W^{(n)}_{x;y} \\
&\simeq \mu\log n + W^{(n)}_{x;y};
\end{aligned}$$

where we let

$$\begin{aligned}
W^{(n)}_{x;y} &= 2\sum_{t=1}^{n-1}\sum_{t'=t+1}^{n}\left[P(\Delta R^{(x)}_t \Delta R^{(x)}_{t'} > 0)\right]^2 \\
&= 2\sum_{t=1}^{n-1}\sum_{t'=t+1}^{n}\left[P(\Delta R^{(x)}_{t'} > 0 \mid \Delta R^{(x)}_t > 0)\right]^2\left[P(\Delta R^{(x)}_t > 0)\right]^2 \\
&= 2\sum_{t=1}^{n-1}\left[P(\Delta R^{(x)}_t > 0)\right]^2\sum_{t'=t+1}^{n}\left[P(\Delta R^{(x)}_{t'-t} > 0)\right]^2.
\end{aligned}$$

Now observing that

$$\sum_{t=1}^{n-1}\Big[P(\Delta R_t^{(x)}>0)\Big]^2\sum_{t'=t+1}^{n}\Big[P(\Delta R_{t'-t}^{(x)}>0)\Big]^2$$
$$=\mu^2\sum_{t=1}^{n-1}t^{-1}(\log n-\log t)$$
$$=\mu^2(\log n)^2-\mu^2\sum_{t=1}^{n-1}t^{-1}\log t$$
$$=\mu^2(\log n)^2-\mu^2\left\{\sum_{t=1}\left(\frac{t}{n}\right)^{-1}\left(\log\frac{t}{n}+\log n\right)\frac{1}{n}\right\}$$
$$=\mu^2(\log n)^2-\mu^2(\log n)^2-\mu^2\left\{\sum_{t=1}\frac{1}{n}\left(\frac{t}{n}\right)^{-1}\log\frac{t}{n}\right\}$$
$$\simeq-\mu^2\int_{1/n}^{1}\frac{\log x}{x}dx;\quad\text{for large enough } n;$$
$$=\frac{1}{2}\mu^2(\log n)^2;$$

it follows:

$$Var\left\{T_{x,y}^{(n)}\right\}\simeq\mu\log n+\mu^2(\log n)^2-\left[E\{T_{x,y}^{(n)}\}\right]^2=\mu\log n.$$

This entails that $\delta_n=O((\log n)^{1/2})$ and

$$(\log n)^{-1}T_{x,y}^{(n)}\xrightarrow{p}1.$$

Our next step in the proof is to show that the events $\left\{\left\{\Delta R_t^{(x)}>0\right\}\cap\left\{\Delta R_t^{(y)}>0\right\}\right\}_t$, or equivalently, the sequence $\left\{1(\Delta R_t^{(x)}>0)1(\Delta R_t^{(y)}>0)\right\}_t$; satisfy some sort of mixing property. This will allows us to invoke a CLT for weakly dependent and heterogenous random variables: From lemma 4 we only need to prove the asymptotic ($j\to\infty$) independence of the events $\left\{\Delta R_{t+j}^{(x)}>0\right\}$ and $\left\{\Delta R_t^{(x)}>0\right\}$.

Moreover, from lemma 2 we have:

$$\lim_{i\to\infty}P\left\{\Delta R_{t+i}^{(x)}>0|\Delta R_t^{(x)}>0\right\}=\lim_{i\to\infty}P\left\{\Delta R_i^{(x)}>0\right\}=0=\lim_{i\to\infty}P\left\{\Delta R_{t+i}^{(x)}>0\right\}.$$

The binary random variables $1(\Delta R_{t+i}^{(x)} > 0)$ and $1(\Delta R_t^{(x)} > 0)$ are thereby asymptotically $(i \to \infty)$ independent and the sequence $\left\{1(\Delta R_t^{(x)} > 0)\right\}_t$ is said to be uniformly mixing. It follows trivially that, under the null hypothesis of independence, the sequence $\left\{1(\Delta R_t^{(x)} > 0)1(\Delta R_t^{(y)} > 0)\right\}_t$ is also uniformly mixing since:

$$
\begin{aligned}
& P\left\{\Delta R_{t+i}^{(x)}\Delta R_{t+i}^{(y)} > 0 | \Delta R_t^{(x)}\Delta R_t^{(y)} > 0\right\} \\
&= P\left\{\Delta R_{t+i}^{(x)} > 0 | \Delta R_t^{(x)} > 0\right\} P\left\{\Delta R_{t+i}^{(y)} > 0 | \Delta R_t^{(y)} > 0\right\} \\
&= \left(P\left\{\Delta R_{t+i}^{(x)} > 0 | \Delta R_t^{(x)} > 0\right\}\right)^2 \to 0 \quad \text{as } i \to \infty; \text{ for any } t,
\end{aligned}
$$

and on the other hand $P\{\Delta R_{t+i}^{(x)} > 0\}P\{\Delta R_{t+i}^{(y)} > 0\} \to 0$, as $i \to \infty$; for any t:

The asymptotic independence of the variables in the partial sum $T_{x;y}^{(n)}$ under the null hypothesis is the first step in proving that a CLT exists for such a sum. To complete the proof we use Bernstein blocking method (see for instance Davidson, 1994). The heuristic reasoning goes as follows. Recalling that

$$
\begin{aligned}
E\left\{1(\Delta R_t^{(x)} > 0)1(\Delta R_t^{(y)} > 0)\right\} &= P\left\{\Delta R_t^{(x)} > 0; \Delta R_t^{(y)} > 0\right\} \\
&= \left(P\left\{\Delta R_t^{(x)} > 0\right\}\right)^2; \text{ under } H_0 \\
&= \mu t^{-1}; \text{ for some positive constant } \mu;
\end{aligned}
$$

we now consider the array of zero-mean random variables defined by:

$$
W_{n;t} = (\sigma^2 \log n)^{-1=2}\left\{1(\Delta R_t^{(x)} > 0)1(\Delta R_t^{(y)} > 0) - \mu t^{-1}\right\}:
$$

Also define $b_n = n^{1-\beta}$ and $r_n = [n/b_n] \sim n^{\beta}$, for some $\beta \in (0,1)$ and with $[\cdot]$ denoting the integer part. We can then write:

$$
\begin{aligned}
S^{(n)}_{x,y} &\triangleq (\sigma^2 \log n)^{-1/2}(T^{(n)}_{x,y} - \mu \log n) \\
&= \sum_{t=1}^{n} W_{n,t} \\
&= \sum_{i=1}^{r_n} Z_{n,i} + W_{n,r_n b_n+1} + \cdots + W_{n,n},
\end{aligned}
$$

with

$$
Z_{n,i} = \sum_{t=(i-1)b_n+1}^{ib_n} W_{n,t}.
$$

Notice that $\sum_{i=1}^{r_n} Z_{n,i}$ contains $b_n r_n$ terms while the sum $W_{n,r_n b_n+1} + \cdots + W_{n,n}$ contains less than b_n terms. Therefore $W_{n,r_n b_n+1} + \cdots + W_{n,n}$ is asymptotically $(n \to \infty)$ negligeable with respect to $\sum_{i=1}^{r_n} Z_{n,i}$, so we can write for large enough n:

$$
S^{(n)}_{x,y} \sim \sum_{i=1}^{r_n} Z_{n,i}.
$$

In order to make the components within the previous sum asymptotically independent, we need to approximate $Z_{n,i}$ by a censored variable $\tilde{Z}_{n,i}$ which we define as:

$$
\tilde{Z}_{n,i} = \sum_{t=(i-1)\tilde{b}_n+1}^{i\tilde{b}_n} W_{n,t},
$$

where $\tilde{b}_n = b_n - \delta_n$, with δ_n denoting any increasing sequence of integers satisfying $\delta_n = o(b_n)$, in such a way that $\sum_{i=1}^{r_n}(Z_{n,i} - \tilde{Z}_{n,i})$ becomes asymptotically $(n \to \infty)$

negligeable with respect to $\sum_{i=1}^{r_n}\tilde{Z}_{n,i}$ (Notice that $\sum_{i=1}^{r_n}(Z_{n,i}-\tilde{Z}_{n,i})$ contains about $r_n\delta_n$ terms, while $\sum_{i=1}^{r_n}\tilde{Z}_{n,i}$ contains $r_n\ell_n$). From the previous discussion one obtains:

$$S_{x,y}^{(n)} \sim \sum_{i=1}^{r_n} \tilde{Z}_{n,i},$$

where now the variables $\left\{\tilde{Z}_{n,i}\right\}_{1\le i\le n}$ are asymptotically ($n \to \infty$) independent. Therefore a CLT can be sought for the standardized sum $S_{xy}^{(n)}$ so that if $s>0$:

$$P\left(S_{x,y}^{(n)} > s\right) \to 1-\Phi(s), \text{ as } n \to \infty.$$

with $\Phi(.)$ denoting the standard Normal distribution function.Now we invoke lemma 6 to prove our weak convergence result under the null hypothesis of independence of the series. This requires showing that the sequence $T_{r_n} = \prod_{i=1}^{r_n}(1+i\lambda Z_{n,i})$ is uniformly integrable for any $\lambda>0$. To avoid this rather burdersome requirement, we can equivalently work out the proof from the truncated series

$$\tilde{Z}_{n,i} = \left(\sum_{t=(i-1)\ell_n+1}^{i\ell_n} W_{n,t}\right) 1\left(\sum_{j=1}^{i-1} Z_{n,k}^2 \le 2\right);$$

by invoking lemma 7. To prove that $\tilde{T}_{r_n}$ is uniformly integrable it is enough then to show that $\sup_n E\left(\max_{1\le i\le r_n} Z_{n,i}^2\right) < \infty$. We have:

$$Z_{n,i}^2 = \left(\sum_{t=(i-1)\ell_n+1}^{i\ell_n} W_{n,t}\right)^2 \sim (\log n)^{-1}\log\frac{i}{i-1} \sim i^{-1}(\log n)^{-1}.$$

Thus $E\left(\max_{1\le i\le r_n} Z^2_{n;i}\right) \sim (\log n)^{-1} < \infty \;\; \forall n$. Our next step is to show that $\sum_{i=1}^{r_n} Z^2_{n;i} \xrightarrow{p} 1$.

$$\sum_{i=1}^{r_n} Z^2_{n;i} = \sum_{i=1}^{r_n}\left(\sum_{t=(i-1)\ell_n+1}^{i\ell_n} W_{n;t}\right)^2$$
$$= \sum_{i=1}^{r_n}\sum_{t=(i-1)\ell_n+1}^{i\ell_n} W^2_{n;t} + 2\sum_{i=1}^{r_n}\sum_{t=(i-1)\ell_n+1}^{i\ell_n-1} W_{n;t}\sum_{t'=t+1}^{i\ell_n} W_{n;t'}.$$

But

$$\sum_{t'=t+1}^{i\ell_n} W_{n;t'} \sim (\log n)^{-1/2}\left[(\log i\ell_n)^{1/2} - (\log t)^{1/2}\right]$$
$$\sim (\log n)^{-1/2}(\log i\ell_n)^{-1/2}\log\frac{i\ell_n}{t}$$
$$= (\log n)^{-1/2}(\log i\ell_n)^{-1/2}\log\left(\frac{i\ell_n}{(i-1)\ell_n + c_n\ell_n}\right); \text{ with } c_n \in (0,1)$$
$$\sim (\log n)^{-1/2}(\log i\ell_n)^{-1/2}\log\left(\frac{i}{i-1}\right); \text{ for large } i$$
$$\sim (\log n)^{-1/2}(\log i\ell_n)^{-1/2} i^{-1}.$$

Therefore

$$\sum_{t=(i-1)\ell_n+1}^{i\ell_n-1} W_{n;t}\sum_{t'=t+1}^{i\ell_n} W_{n;t'}$$
$$\sim (\log n)^{-1}\left(\log i\ell_n\right)^{-1/2} i^{-1}\left[(\log i\ell_n)^{1/2} - (\log(i-1)\ell_n)^{1/2}\right]$$
$$\sim (\log n)^{-1}\left(\log i\ell_n\right)^{-1} i^{-2}.$$

Finally,

$$\sum_{i=1}^{r_n} Z_{n;i}^2 = \sum_{i=1}^{r_n}\sum_{t=(i-1)\tilde{s}_n+1}^{i\tilde{s}_n} W_{n;t}^2 + 2\sum_{i=1}^{r_n}\sum_{t=(i-1)\tilde{s}_n+1}^{i\tilde{s}_n-1} W_{n;t}\sum_{t'=t+1}^{i\tilde{s}_n} W_{n;t'}$$

$$\approx (\log n)^{-1}\sum_{i=1}^{r_n}\left[(\log i\tilde{s}_n) - (\log(i-1)\tilde{s}_n)\right] + (\log n)^{-1}\sum_{i=1}^{r_n}\left(\log i\tilde{s}_n\right)^{-1} i^{-2}$$

$$\approx (\log n)^{-1}\sum_{i=1}^{r_n}\log\frac{i}{i-1}$$

$$\approx (\log n)^{-1}\sum_{i=1}^{r_n} i^{-1}$$

$$\approx (\log n)^{-1}\log r_n = \beta.$$

Another condition that needs being proved is that $\max_{1\le i\le r_n}|\tilde{Z}_{n;i}| \xrightarrow{p} 0$. This can be obtained by noting that

$$\tilde{Z}_{n;i} \le \sum_{t=(i-1)\tilde{s}_n+1}^{i\tilde{s}_n} W_{n;t} \approx (\log n)^{-1=2}\left[(\log i\tilde{s}_n)^{1=2} - (\log(i-1)\tilde{s}_n)^{1=2}\right]$$

$$\approx (\log n)^{-1=2}(\log i\tilde{s}_n)^{-1=2}\log\frac{i}{i-1}$$

$$\approx (\log n)^{-1=2}(\log i\tilde{s}_n)^{-1=2} i^{-1};$$

Thus

$$\max_{1\le i\le r_n}|\tilde{Z}_{n;i}| \approx (\log n)^{-1=2}(\log\tilde{s}_n)^{-1=2} \to 0 \text{ as } n\to\infty.$$

The last condition which must be checked before concluding from direct application of lemmas 6 and 7 is that $E(\tilde{T}_{r_n}) \to 1$ as $n \to \infty$. This follows straightforwardly from the asymptotic independence of the zero-mean variables $\tilde{Z}_{n;i}$. Indeed,

$$E(\tilde{T}_{r_n}) = E\left\{\prod_{i=1}(1+i\lambda\tilde{Z}_{n;i})\right\} \to \prod_{i=1}\left[1+i\lambda E(\tilde{Z}_{n;i})\right] = 1.$$

∎

Appendix A2

Let $y_t \sim I(2)$ and $x_t \sim I(1)$ be independent series. Since

$$P(\Delta R_t^{(y)} > 0) = O(1)$$

$$P(\Delta R_t^{(x)} > 0) = O(t^{-1/2});$$

we get

$$\begin{aligned}
E\left\{T_{y;x}^{(n)}\right\} &= \sum_{t=1} P(\Delta R_t^{(y)} > 0)P(\Delta R_t^{(x)} > 0) \\
&\sim \sum_{t=1} t^{-1/2}; \quad \text{since } P(\Delta R_t^{(y)} > 0) = 1 \\
&\sim n^{1/2} \sum_{t=1} \left(\frac{t}{n}\right)^{-1/2} \frac{1}{n} \\
&\sim n^{1/2} \int_0^1 x^{-1/2} dx \\
&\sim n^{1/2}.
\end{aligned}$$

Therefore

$$(\log n)^{-1/2} E(T_{y;x}^{(n)} - \mu \log n) = O((\log n)^{-1/2} n^{1/2}) \to \infty; \quad \text{as } n \to \infty.$$

As for the variance:

$$Var\left\{(\log n)^{-1/2} T_{y;x}^{(n)}\right\} = (\log n)^{-1} E\left\{\left[T_{y;x}^{(n)}\right]^2\right\} - (\log n)^{-1}\left[E\left\{T_{y;x}^{(n)}\right\}\right]^2$$

and since

$$\left[E\left(T_{y;x}^{(n)}\right)\right]^2 \sim n$$

$$E\left\{\left[T_{y;x}^{(n)}\right]^2\right\} = \sum_{t=1}^{n} P(\Delta R_t^{(y)} > 0)P(\Delta R_t^{(x)} > 0) + 2\sum_{t=1}^{n-1}\sum_{t'=t+1}^{n} P(\Delta R_t^{(x)}\Delta R_{t'}^{(x)} > 0)P(\Delta R_t^{(y)}\Delta R_{t'}^{(y)} > 0)$$

$$= \sum_{t=1}^{n} P(\Delta R_t^{(y)} > 0)P(\Delta R_t^{(x)} > 0)$$

$$+2\sum_{t=1}^{n-1} P(\Delta R_t^{(x)} > 0)P(\Delta R_t^{(y)} > 0)\sum_{t'=t+1}^{n} P(\Delta R_{t'}^{(x)} > 0 | \Delta R_t^{(x)} > 0)P(\Delta R_{t'}^{(y)} > 0 | \Delta R_t^{(y)} > 0)$$

$$\sim \sum_{t=1}^{n} P(\Delta R_t^{(x)} > 0) + 2\sum_{t=1}^{n-1} P(\Delta R_t^{(x)} > 0)\sum_{\tau=1}^{n-t} P(\Delta R_\tau^{(x)} > 0)$$

$$\sim n^{1/2} + \sum_{t=1}^{n-1} t^{-1/2}(n-t)^{1/2}$$

$$\simeq n^{1/2} + n\int_0^1 \sqrt{\frac{1-x}{x}}dx; \text{ for large } n$$

$$\sim n^{1/2} + n$$

Finally, we get:

$$Var\left((\log n)^{-1/2}T_{y;x}^{(n)}\right) = O((\log n)^{-1}n^{1/2}) \to \infty; \quad \text{as } n \to \infty$$

$$\sqrt{Var\left\{(\log n)^{-1/2}(T_{y;x}^{(n)} - \mu\log n)\right\}} = O((\log n)^{-1/2}n^{1/4}) = o\left((\log n)^{-1/2}E(T_{y;x}^{(n)} - \mu\log n)\right);$$

which tells us that the standard deviation of our standardised CRCC statistic grows more slowly than its mean. Therefore

$$(\log n)^{-1/2}(T_{y;x}^{(n)} - \mu\log n) \xrightarrow{p} \infty; \quad \text{as } n \to \infty.$$

Now let z_t be an $I(0)$ process independent of x_t. We have:

$$E(T_{z;x}^{(n)}) = \sum_{t=1}^{n} P(\Delta R_t^{(z)} > 0)P(\Delta R_t^{(x)} > 0).$$

And for t large enough:

$$P(\Delta R_t^{(z)} > 0) = O(t^{-1}),$$

$$P(\Delta R_t^{(x)} > 0) = O(t^{-1/2}).$$

Therefore:

$$E(T_{z;x}^{(n)}) = O\left(\sum_{t=1}^{n} t^{-3/2}\right)$$

$$(\log n)^{-1/2}E(T_{z;x}^{(n)} - \mu \log n) = O\left((\log n)^{-1/2}\left(\sum_{t=1}^{n} t^{-3/2} - \mu \log n\right)\right).$$

But $\sum_{t=1}^{n} t^{-3/2} = O(1)$, and consequently:

$$E(T_{z;x}^{(n)}) = O(1)$$

$$(\log n)^{-1/2}E(T_{z;x}^{(n)} - \mu \log n) \to -\infty \text{, as } n \to \infty.$$

On the other hand,

$$\begin{aligned}
E\left\{(T_{z;x}^{(n)})^2\right\} &= \sum_{t=1}^{n} P(\Delta R_t^{(z)} > 0)P(\Delta R_t^{(x)} > 0) + 2\sum_{t=1}^{n-1}\sum_{t'=t+1}^{n} P(\Delta R_t^{(x)}\Delta R_{t'}^{(x)} > 0)P(\Delta R_t^{(z)}\Delta R_{t'}^{(z)} > 0) \\
&\sim \sum_{t=1}^{n} t^{-3/2} + 2\sum_{t=1}^{n-1} P(\Delta R_t^{(x)} > 0)P(\Delta R_t^{(z)} > 0)\sum_{t'=t+1}^{n} P(\Delta R_{t'}^{(x)} > 0|\Delta R_t^{(x)} > 0)P(\Delta R_{t'}^{(z)} > 0|\Delta R_t^{(z)} > 0) \\
&\sim \sum_{t=1}^{n} t^{-3/2} + 2\sum_{t=1}^{n-1} t^{-3/2}\sum_{\tau=1}^{n-t} P(\Delta R_\tau^{(x)} > 0)P(\Delta R_\tau^{(z)} > 0) \\
&\sim \sum_{t=1}^{n} t^{-3/2} + 2\sum_{t=1}^{n-1} t^{-3/2}\sum_{\tau=1}^{n-t} \tau^{-3/2} = O(1).
\end{aligned}$$

And finally, for the variance:

$$Var\left\{(\log n)^{-1/2}T^{(n)}_{z;x}\right\} \sim (\log n)^{-1} \to 0, \text{ as } n \to \infty:$$

Accordingly from lemma 5:

$$(\log n)^{-1/2}(T^{(n)}_{z;x} - \mu \log n) \xrightarrow{p} -\infty:$$

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